Correction to "Variable-to-Fixed Length Codes are Better than Fixed-to-Variable Length Codes for Markov Sources"

J. Ziv, *Fellow, IEEE*

S. De Agostino and M. Cohn from Brandeis University called attention to a flaw in Theorem 2 of the above correspondence.[1]

The following corrections should be inserted on p. 863.

Theorem 2: For every $M > \beta / \min P(X_1^K)$ where $\beta \triangleq 1/\min_{\boldsymbol{A}^{K+1}} [P(X_1^{K+1})/P(X_1^K)]$, there exists a VFL code with no more than M codewords such that

$$\rho_c \leq \rho(\infty) \frac{\log M}{\log M - \log \beta - I_K}. \tag{20}$$

Proof: b) replace α/M by β/M.

Clearly, there are no more than M codewords in the code tree. At the same time, by construction, no leaf has a probability less than β/M. Thus,

$$H_c = -E \log P(X_j^c) \geq \log M/\beta = \log M - \log \beta. \tag{21}$$

Therefore, by (16), (17), and (21),

$$\rho_c \leq \lim_{n\to\infty} \frac{Eq_c(\boldsymbol{X}) \log M}{n \log \alpha} \leq \rho(\infty) \frac{\log M}{\log M - \log \beta - I_K}. \qquad \square$$

In conclusion, it is clear from Theorem 1 and Theorem 2 that when $I_K \gg \log \beta$, ρ_c for the VFL code of Theorem 2 approaches the lower bound in Theorem 1, namely

$$\rho_c(\mathrm{VFL}) \approx \rho(\infty) \frac{1}{1 - I_K / \log M}.$$

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The author is with AT&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, NJ 07974. He is on sabbatical leave from the Technion in Israel Institute of Technology, Haifa 32000, Israel.

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[1] J. Ziv. *IEEE Trans. Inform. Theory,* vol. 36, pp. 861–863, July 1990.

At the same time, we have for the best FVL code that

$$\rho_c(\mathrm{FVL}) \geq \rho(\infty) \frac{1}{1 - I_K / H_c}$$

and since $H_c < \log M$, its performance is inferior to that of the VFL code of Theorem 2.

If $\beta \leq \alpha^2$, the conclusion holds whenever $I_K \gg 2 \log \alpha$. If $\beta > \alpha^2$, a dithering sequence $\boldsymbol{N}$ may be added to $\boldsymbol{X}$ modulo $\boldsymbol{A}$ prior to coding, and subtracted from the output of the decoder, where $\boldsymbol{N}$ is a realization of an i.i.d. process with

$$P_r(N_i = 0) = 1 - \frac{1}{\alpha}; \qquad i = 1, 2, \cdots$$

$$P_r(N_i = x) = \frac{1}{\alpha(\alpha - 1)}; \qquad \text{any } x \neq 0; \qquad x \in \boldsymbol{A};$$

$$\boldsymbol{A} = [0, 1, \cdots, \alpha - 1].$$

Let $\tilde{\boldsymbol{X}} = \boldsymbol{X} \oplus \boldsymbol{N}$.

Then, it follows from the derivation of (17) that

$$\rho_c = \lim_{n\to\infty} \frac{Eq_c(\boldsymbol{X}, \boldsymbol{N})}{n \log \alpha} EL(\tilde{\boldsymbol{X}}_i^c)$$

$$\leq \frac{H + 2h(\frac{1}{\alpha})}{\log \alpha} \frac{\log M}{\tilde{H} + I_K},$$

where

$$\tilde{H}_c = -E \log P_r(\tilde{\boldsymbol{X}}_i^C)$$

and where

$$h(\epsilon) = -\epsilon \log \epsilon - (1 - \epsilon) \log(1 - \epsilon).$$

Also

$$\tilde{\beta} = \min_{\tilde{\boldsymbol{X}}_{-L}^0} P_r(\tilde{X}_1 | \tilde{\boldsymbol{X}}_{-L}^0) > \frac{1}{\alpha^2}, \quad \text{for every } L = 0, 1, 2, \cdots.$$

Therefore, it follows that the conclusion holds whenever

$$I_K \gg 2 \log \alpha + \frac{2h(\frac{1}{\alpha})}{\rho(\infty) \log \alpha}.$$

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